



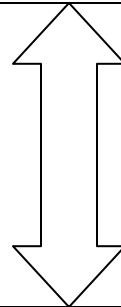
Simultaneous Modeling of Liquid and Gaseous Phases in Heap Leaching for Copper Production

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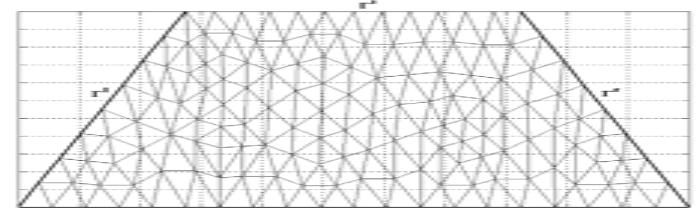
HEAP LEACHING



Theory of Porous Media

IMMISCIBLE FLOW	REACTIVE TRANSPORT
Phase 1: solution. Phase 2: gas.	Component 1: copper's ions. Component 2: sulfuric acid. Component 3: oxygen. Component 4: air. ⋮

Two-Phase Flow System



$$\frac{\partial(\phi \rho_\alpha s_\alpha)}{\partial t} + \operatorname{div}(\rho_\alpha \vec{v}_\alpha) = \rho_\alpha q_\alpha$$

$$\vec{v}_w = -\frac{k_{r\alpha}}{\mu_\alpha} k (\nabla p_\alpha - \rho_\alpha \vec{g})$$

$$p_c(s_w) = p_n - p_w$$

$$s_w + s_n = 1$$

Initial and Boundary Conditions

$$\begin{aligned} s_w(\vec{x}, t) &= s_w^o , & \vec{x} \in \Omega & \quad t = 0 \\ p_n(\vec{x}, t) &= p_A , & \vec{x} \in \Omega, & \quad t = 0 \\ (\vec{v}_w \cdot \vec{n})(\vec{x}, t) &= -R , & \vec{x} \in \Gamma^i & \quad t > 0 \\ (\vec{v}_w \cdot \vec{n})(\vec{x}, t) &= 0 , & \vec{x} \in \Gamma^r \cup \Gamma^l & \quad t > 0 \\ (\nabla p_w \cdot \vec{n})(\vec{x}, t) &= 0 , & \vec{x} \in \Gamma^o & \quad t > 0 \\ (\nabla s_n \cdot \vec{n})(\vec{x}, t) &= 0 , & \vec{x} \in \Gamma^r \cup \Gamma^l \cup \Gamma^i & \quad t > 0 \\ (\vec{v}_n \cdot \vec{n})(\vec{x}, t) &= 0 , & \vec{x} \in \Gamma^o & \quad t > 0 \end{aligned}$$

Brooks-Corey & van Genuchten

	$p_c(S_w)$	$k_{rw}(S_w)$	$k_{rn}(S_w)$
BC	$p_d S_e^{-1/\lambda}$	$S_e^{\frac{2+3\lambda}{\lambda}}$	$(1-S_e)^2(1-S_e^{\frac{2+\lambda}{\lambda}})$
VG	$\frac{1}{\alpha} (S_e^{-1/m} - 1)^{1/n}$	$S_e^\varepsilon (1 - (1 - S_e^{\frac{1}{m}})^m)^2$	$(1-S_e)^\gamma (1-S_e^{\frac{1}{m}})^{2m}$

$$\lambda(m) = \frac{m}{1-m} (1 - 2^{-1/m})$$

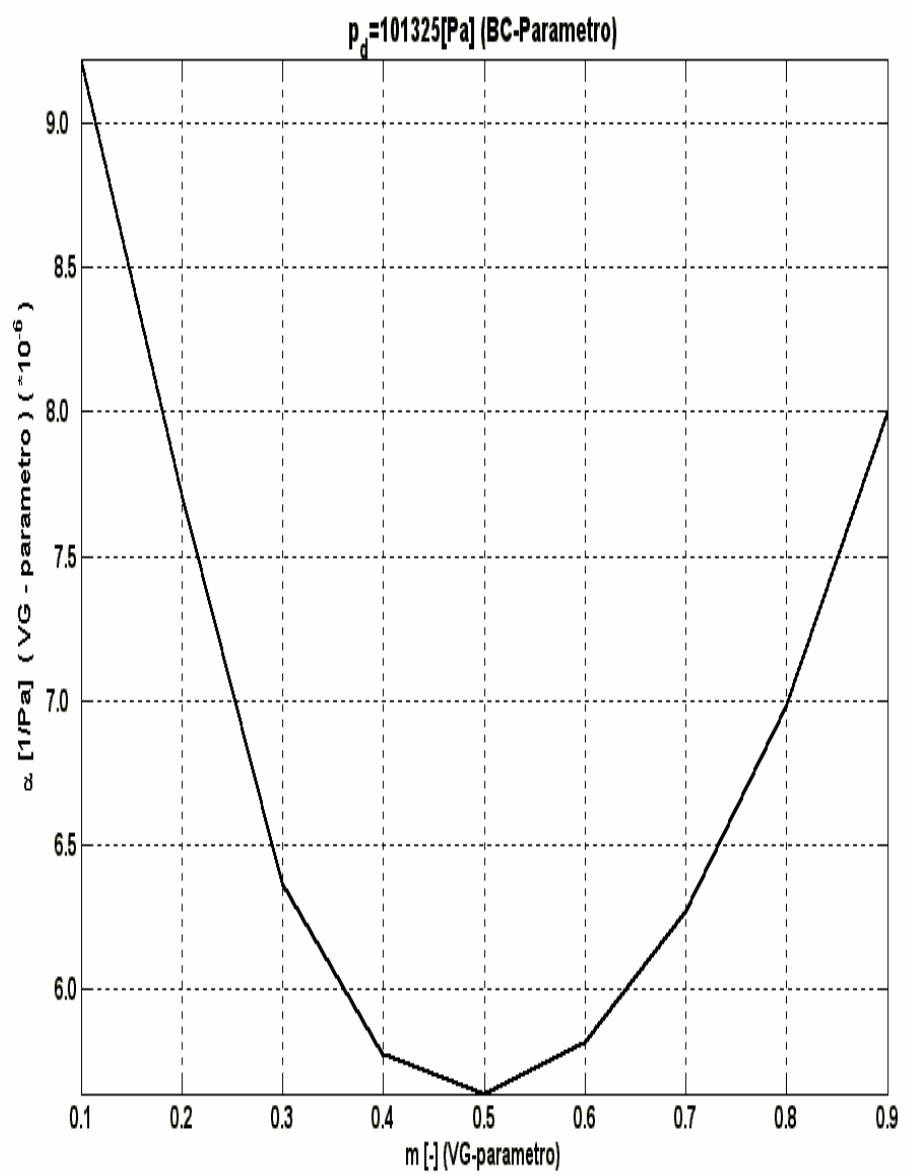
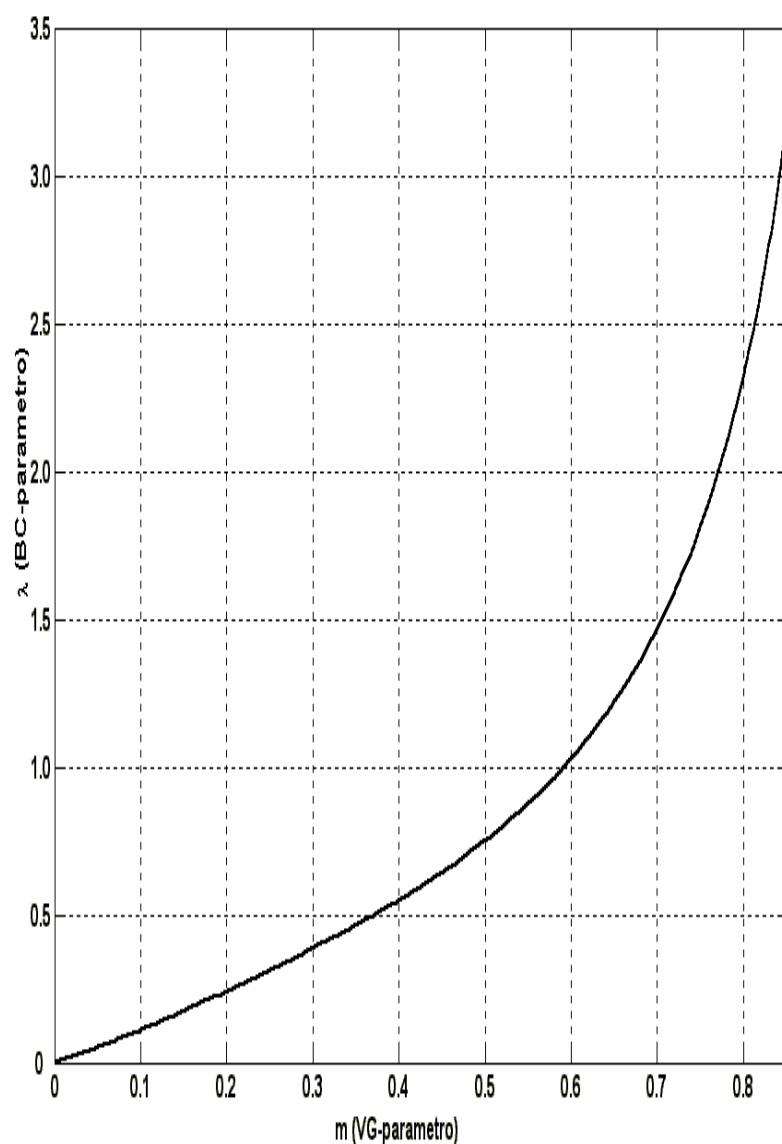
$$\bar{S}_x(m) = 0.72 - 0.35 e^{-(1-m)^{-4}}$$

BC: $p_d > 0, 0.2 < \lambda < 3.0.$

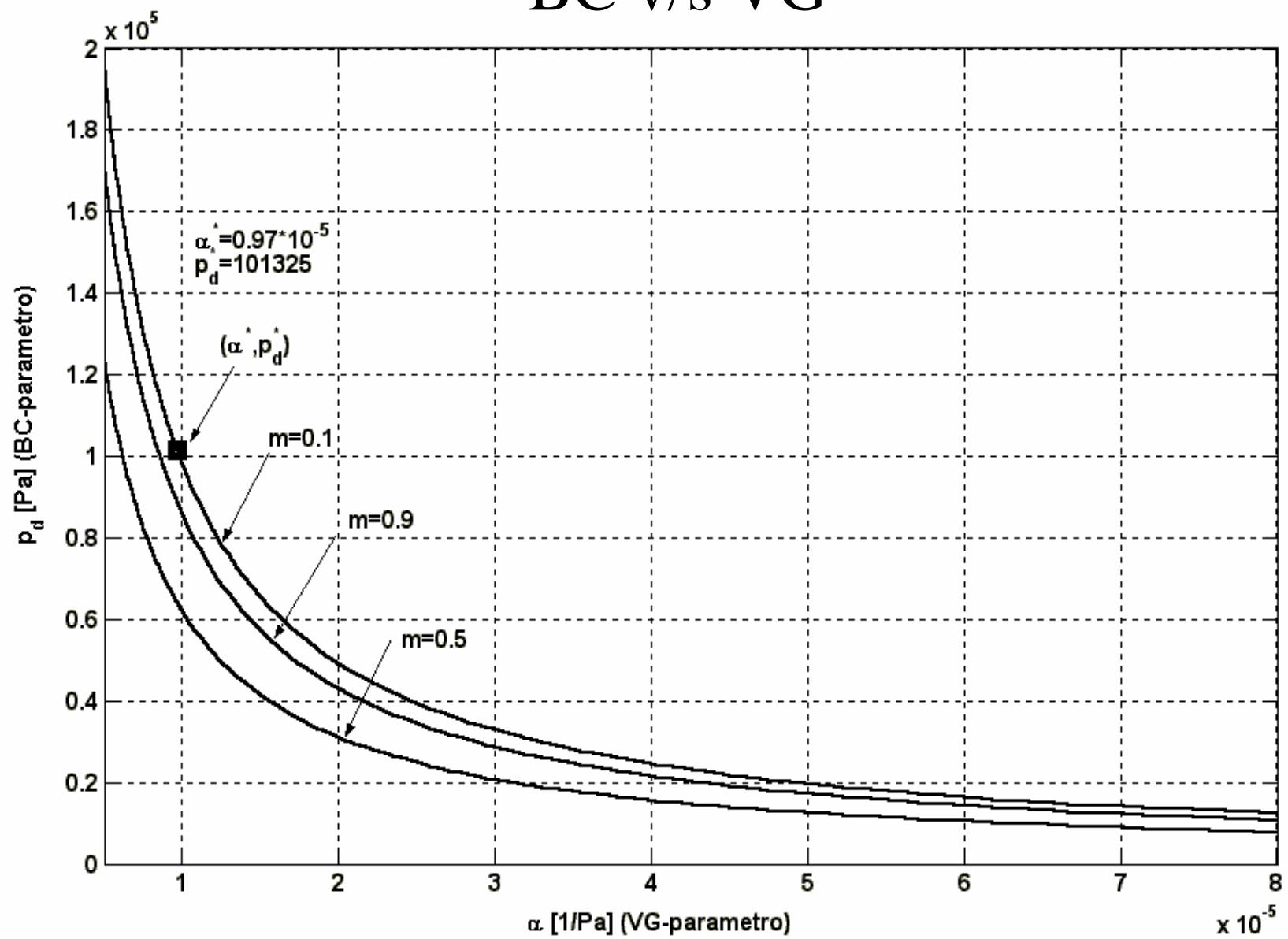
VG: $\varepsilon, \gamma, 0 < m < 1, m = 1 - 1/n, \alpha > 0.$

$$p_d(m, \alpha) = \frac{\bar{S}_x(m)}{\alpha} (\bar{S}_x^{-1/m}(m) - 1)^{1-m}$$

BC v/s VG



BC v/s VG



Numerical Solution

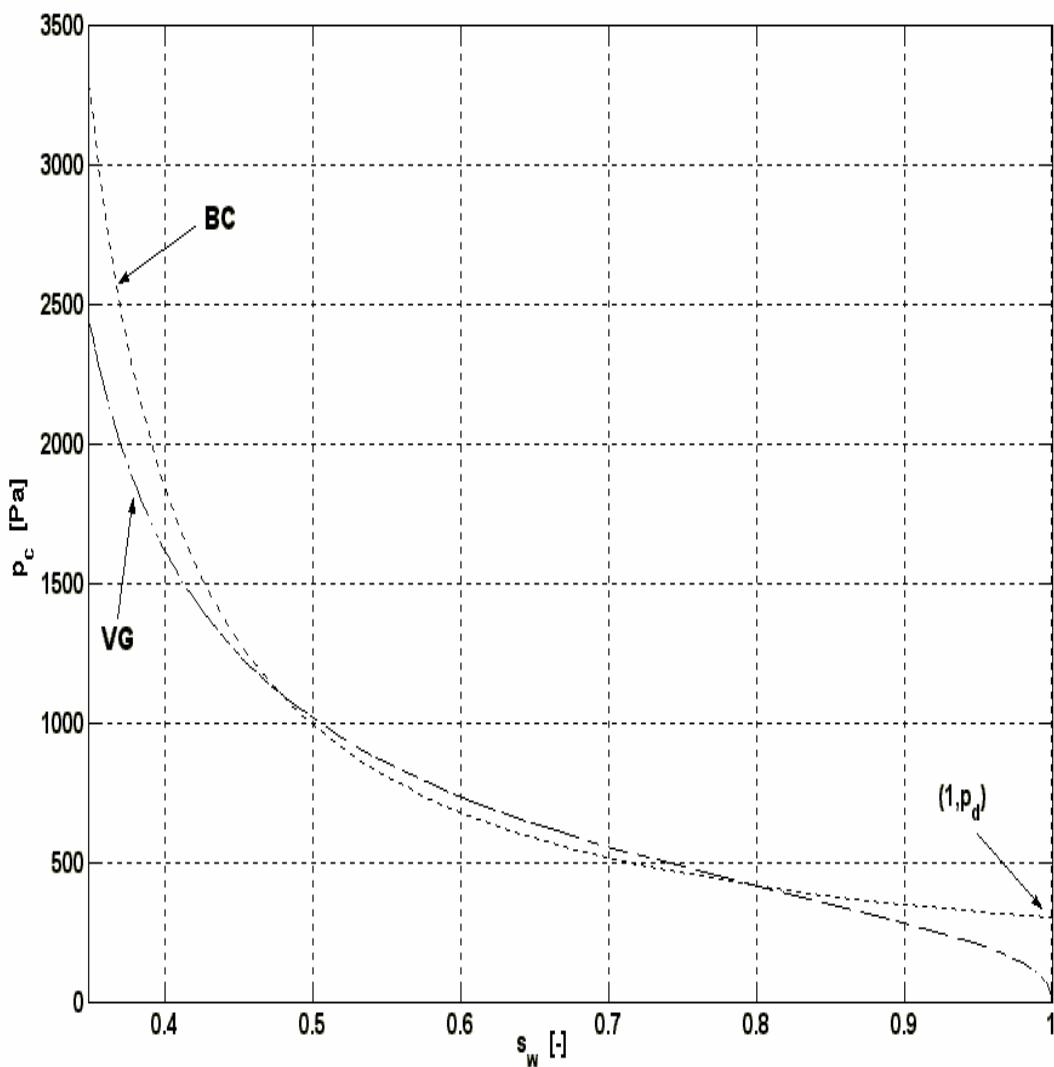
- Fractional Flow Formulation.
 - Saddle Point Problem + Conservation Law.
- Mixed Finite Element.
 - Aproximation of Darcy's flows and pressures.
- Finite Volume.
 - Aproximation of liquid phase's saturation.

E. Cariaga & F. Concha & M. Sepulveda

Flow through porous media with applications to heap leaching of copper ores

Chemical Engineering Journal, vol. 111, 2-3, pp. 151-165, 2005.

CASE 1



$$k = 6.23 \cdot 10^{-11} [m^2]$$

$$\phi = 0.459 [-]$$

$$\mu_w = 9 \cdot 10^{-4} [kg/ms]$$

$$\mu_n = 1.85 \cdot 10^{-5} [kg/ms]$$

$$\rho_w = 1011 [kg/m^3]$$

$$\rho_n = 1.16 [kg/m^3]$$

$$\lambda = 1 [-]$$

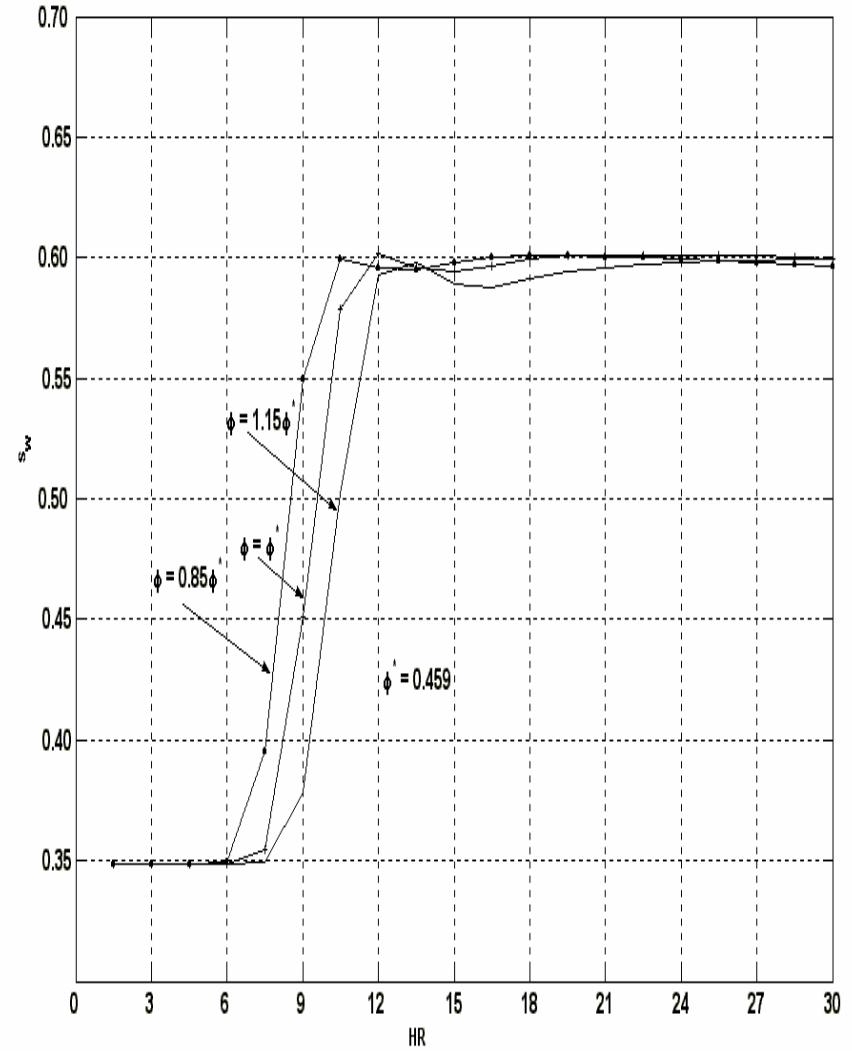
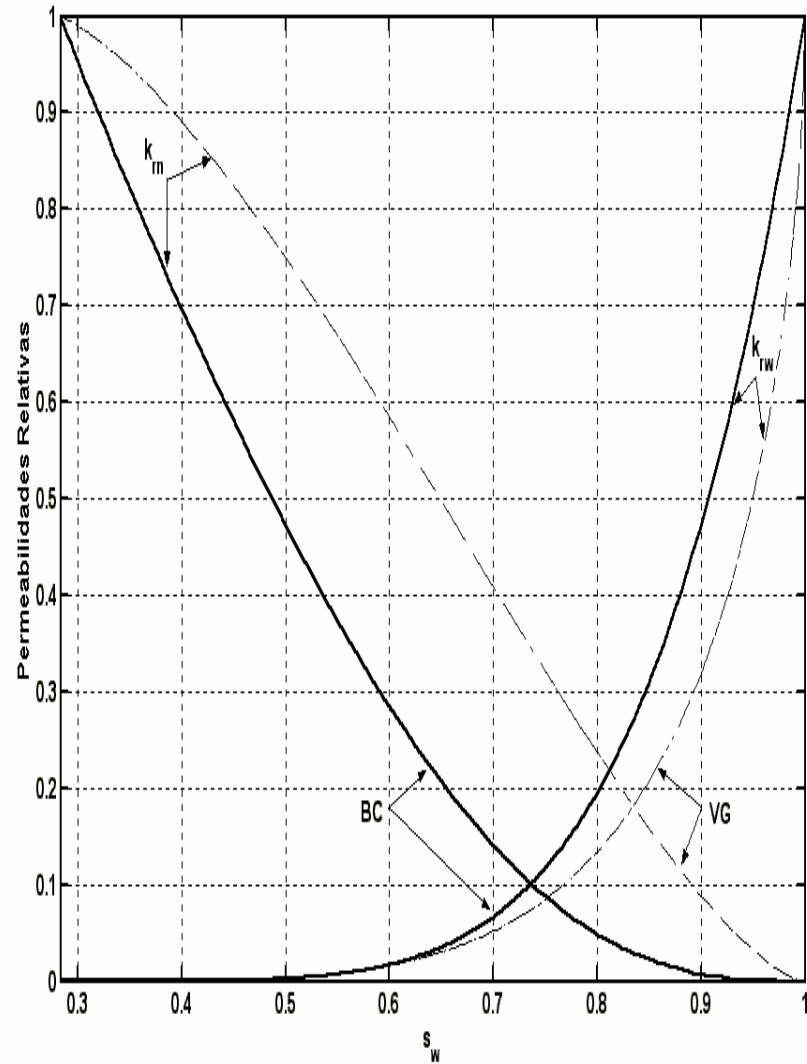
$$p_d = 300 [Pa]$$

$$s_{nr} = 0 [-]$$

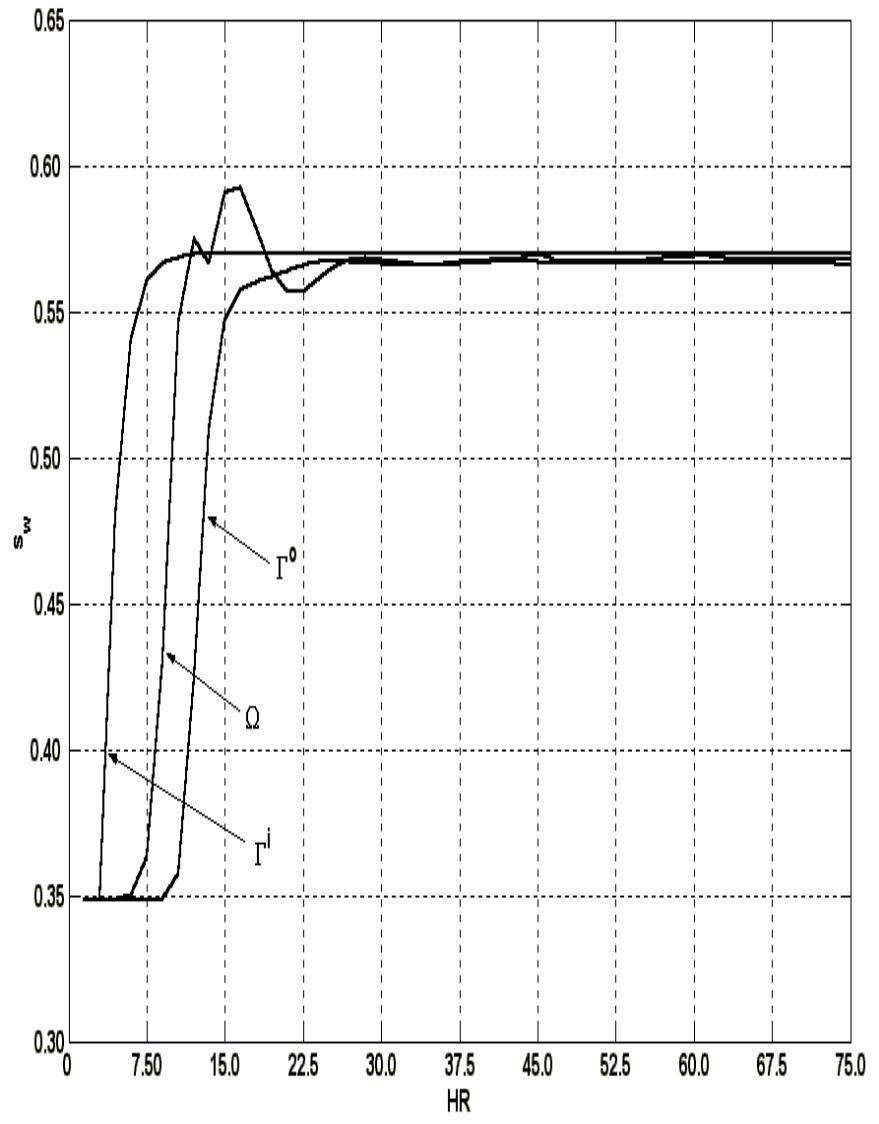
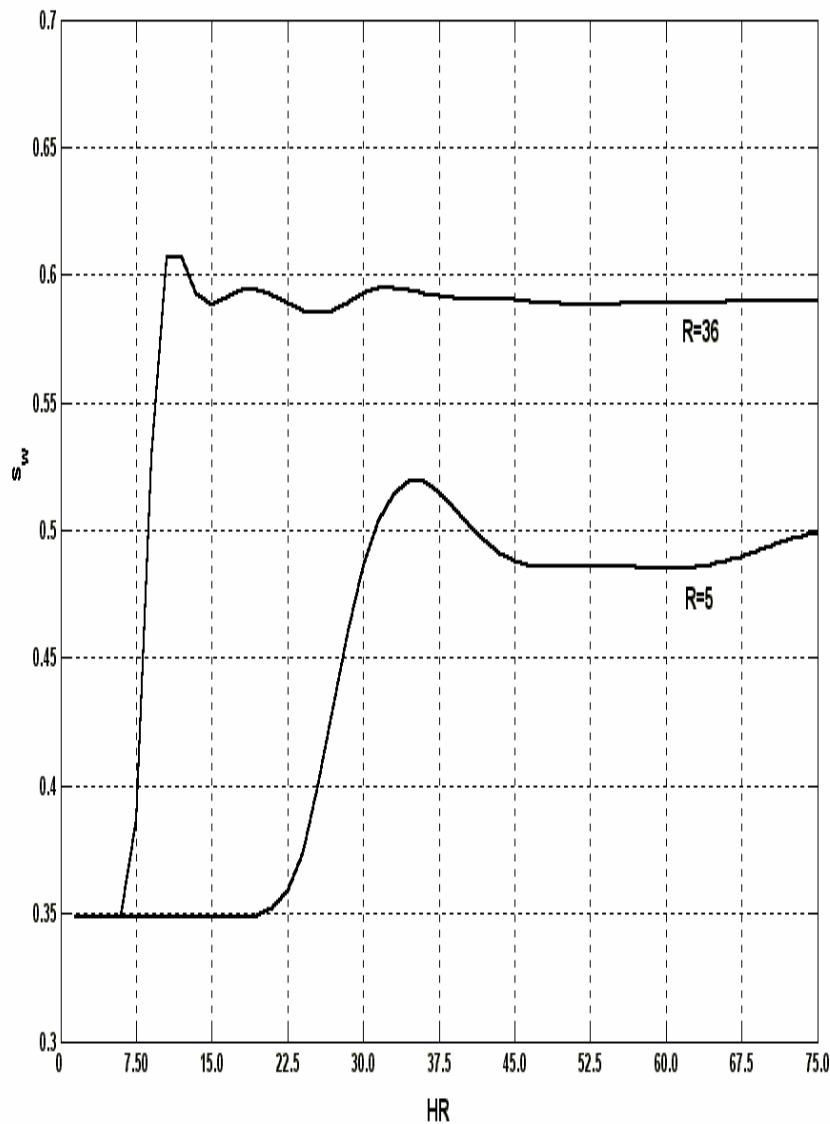
$$s_{wr} = 0.283 [-]$$

$$\vec{s}_w(x, t=0) = 0.348 [-]$$

...CASE 1...



...CASE 1...



CASE 2

$t = 48, 60, 72, 94 \text{ (h)}$

$$k = 1.78 \cdot 10^{-12} [m^2]$$

$$\phi = 0.33 [-]$$

$$\mu_w = 10^{-3} [kg/ms]$$

$$\mu_n = 1.85 \cdot 10^{-5} [kg/ms]$$

$$\rho_w = 1011 [kg/m^3]$$

$$\rho_n = 1.165 [kg/m^3]$$

$$\alpha = \frac{1.368 * 10.34}{101325} [1/Pa]$$

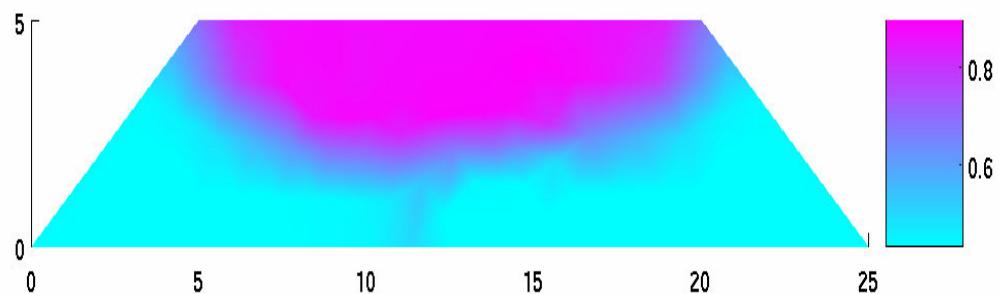
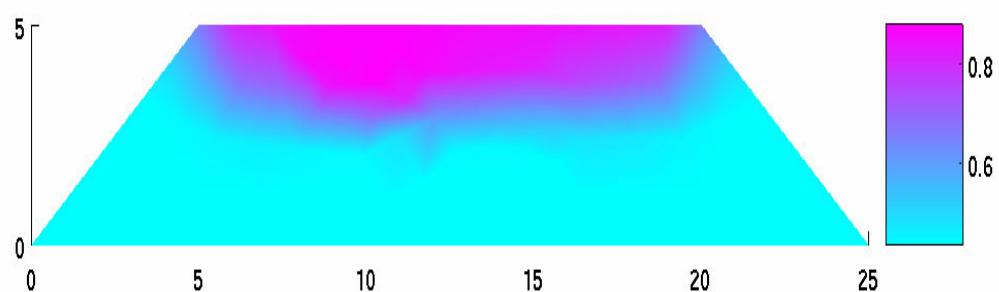
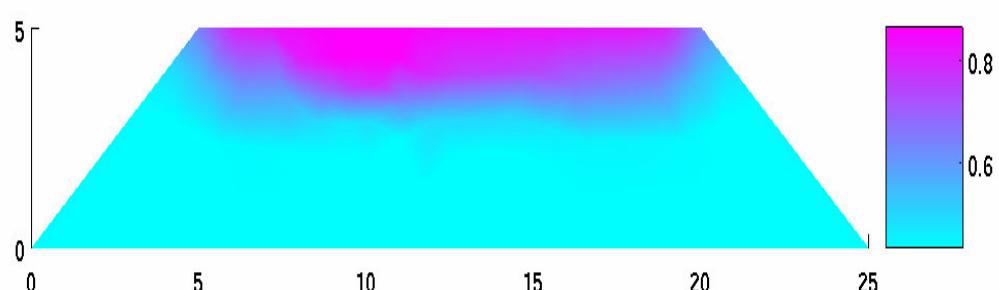
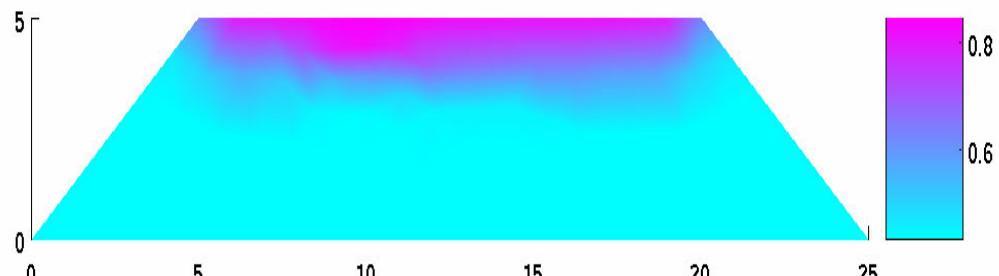
$$n = 1.411 [-]$$

$$s_{nr} = 0 [-]$$

$$s_{wr} = 0 [-]$$

$$\vec{s}_w(x, t=0) = 0.4343 [-]$$

$$R = 3.6 [L/h/m^2]$$



Conclusions & Remarks

- Heap Leaching: Theory of Porous Media.
 - Flow + Transport: Coupled.
 - Interaction between phases liquid and gaseous.
 - Interchange of components between phases.
- BC and VG: ζ estimates of parameters ?.
- Only Finite Volume to two phase system.
 - R. Helmig (1997), University of Stuttgart, Germany.
- Future work:
 - 3D simulations for flow and transport.
 - Porosity and absolute permeability: not constants.
 - Non-isothermal system.
 - Heterogeneous porous medium.
 - ...

NOMENCLATURE

$\alpha,$	phase
$\alpha = w,$	liquid phase
$\alpha = n,$	gaseous phase
$\rho_\alpha,$	density [kg/m^3]
$s_\alpha,$	saturation [-]
$S_e,$	efective saturation [-]
$s_{r\alpha},$	residual saturation [-]
$\theta_\alpha = \phi s_\alpha,$	liquid's content [-]
$\vec{v}_\alpha,$	Darcy's flow [m/s]
$p_\alpha,$	pressure [Pa]
$k_{r\alpha},$	relative permeability [-]
$\mu_\alpha,$	viscosity [kg/m/s]
$\lambda_\alpha,$	mobility [ms/kg]
$\phi,$	porosity [-]
$k,$	absolute permeability [m^2]
$\vec{n},$	unitary normal vector
$\Omega,$	mathematical domain 2D
$\Gamma,$	boundary
$R,$	Irrigation's ratio [L/h/m^2]
$t,$	time [s]
$\vec{x},$	spatial point
$p_c,$	capillary's pressure [Pa]
$m, n, \alpha, \varepsilon, \gamma,$	parameters VG
$\lambda, p_d,$	parámetros BC
$q_\alpha,$	source
$g,$	gravity
$p_A,$	atmospheric pressure
$s_w^o,$	initial saturation
$\bar{S}_x,$	match - point saturation